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Euclid: Reception in the Renaissance

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Abstract

Although the Latin Middle Ages received a number of versions of *Euclid's Elements* and several other Euclidean works, by the fourteenth century, only the Campanus redaction from c. 1259 was in circulation. In the fourteenth and fifteenth century, this redaction was encountered by students of Arts or Medicine university faculties, even though we have scant evidence that Euclid impressed their minds. In the fifteenth century, other circles discovered him: Alberti took over the idea of elements, Regiomontanus used Euclid alongside Archimedes as an argument for the superiority of mathematics over philosophy, and one Florentine abacus school tradition was able to give correct references to the *Elements*.

A turn arrived with book printing. In 1482, the Campanus *Elements* were printed, and in 1498 and 1501, Giorgio Valla inserted pseudo-Euclidean and Euclidean material in two bulky volumes. A new though somewhat problematic Latin translation from the Greek (including also some minor works) was published by Zamberti in 1505, and until 1540 a number of reprints or reeditions of Campanus's and Zamberti's texts were published – at times in combination. From the 1540s onward, revisions, selections, and vernacular translations began to appear, all based on the same two texts. In 1572, however, Commandino made a new Latin translation from Zamberti's text and a sounder manuscript, and in 1574 Clavius produced a didactically oriented redaction. These two set the scene for the next two centuries.

Medieval Latin Background

Until the beginnings of the twelfth century, the Latin Middle Ages had access to Euclid only through Boethian translation of the *Elements* (or, quite likely, of an epitome of that work) – another translation, also from c. 500 CE (Bohlin 2012), seems not to have circulated. Part of the Boethian translation was conserved in coherent form – how much and for how long is disputed; Boethian fragments were also integrated in gromatic writings (which served didactical purposes rather than surveying).

In the twelfth century, a new translation directly from the Greek was made (ed. Busard 1987); it was used occasionally by Fibonacci (Folkerts 2006, IX) but left few traces beyond that. The translations from the Arabic made by Gerard of Cremona (ed. Busard 1984) and Hermann of Carinthia (ed. Busard 1967) also had limited circulation. The one prepared by

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Adelard of Bath, so-called Adelard I, on the other hand, became the basis for the didactically adapted redaction known as Adelard II (ed. Busard and Folkerts 1992), probably due to Robert of Chester (or "of Ketton"), and for the epistemologically more sophisticated "Adelard III" (ed. Busard 2001), probably the work of John of Tynemouth. Versions II and III were then used by Campanus of Novara for his redaction, written between 1255 and 1259 (ed. Busard 2005). This redaction was well adapted to use in the scholastic university, where at least part of the *Elements* was supposed to enter the Arts curriculum (in some places alternatively with Book I of Witelo's *Perspectiva*). Campanus *Elements* came to dominate until well into the sixteenth century. All of these except the Hermann translation contain 15 books, that is, they include a Book XIV written by Hypsicles and a Book XV written by Isidore of Miletus; of the Hermann translation, only 12 books are extant.

Euclid's *Data* and *Optics* were also translated in the twelfth century (from the Greek as well as the Arabic) and so was the pseudo-Euclidean *Catoptrics* (Murdoch 1971, 444). The (probably pseudo-)Euclidean *De ponderoso et levi* (ed. trans. Moody and Clagett 1952, 21–31) was translated from the Arabic. However, none of these works had an influence coming close to that of the *Elements*.

Early Humanist Interest and Knowledge

The fourteenth-century humanists who had frequented one of the integrated Arts and Medicine faculties of Italian universities or an Arts or a Medicine faculty elsewhere were likely to have gained some familiarity with the first books of the *Elements*, but sources give little more than hints (cf. Siraisi 1973, 74–77). A copy of the Campanus version in the library of S. Spirito in Florence listed in a catalogue from 1451 *may* have been part of the legacy from Boccaccio (Ullman 1964, 285). In any case, in his *Della genealogia de gli Dei*, we find an attack on those who have had brief contact with the schools ("have seen its door") and have looked into some vernacular booklets and now wish to be held philosophers; they are characterized by citing authors they have never seen – "Priscian, Aristotle, Cicero, Aristarchus, Euclid, Ptolemy, and others, most famous in the sciences" (Boccaccio 1564, 225^{r-v}). This may be seen as a minimal list of those authors people of manners were supposed to know about.

In the fifteenth century, Leon Battista Alberti went beyond name-dropping, manuscript possession, and possibly manuscript reading. His *Elementi de pittura* (ed. Grayson 1973, 109–129) from c. 1435 (as well as the parallel Latin *Elementa*) not only borrow the Euclidean title. "For the sake of brevity," they also open with 5 sets of definitions, 22 in total in each of the 2 versions (not fully identical, however); for the rest they consist of specified elementary tasks similar to Euclidean problems – firstly "to describe a straight line from one point to another one." As stated in the opening of *De pictura* (ed. Grayson 1973, 6–107), Alberti does so borrowing from mathematicians but adapting what they say about the merely intelligible to a topic interested only in that which can be seen. This is probably the first Renaissance example of use of the "geometric method" – inspired by Euclid and not by Archimedes, whose axiomatic-deductive works Alberti does not know.

In 1564, Regiomontanus – first trained and active in the University of Vienna and then under the influence of Bessarion drawn into Italian Humanism – held a series of lectures on the astronomer al-Farghānī in Padua. The inaugural lecture (ed. Schmeidler 1972, 43–53), an oration "explaining the mathematical sciences and their utility," mixes the two currents of thought (Byrne 2006) but also shows traces of that pride of Italian mathematical practitioners which made them claim priority of mathematics over philosophy. Regiomontanus uses Euclid and Archimedes for that purpose – while philosophy is split into warring schools, "Euclid's theorems have the same certitude today as a thousand years ago, and Archimedes's inventions will call forth no less

admiration in a thousand centuries than pleasure in us when reading them." In the beginning of the oration, Euclid has already been declared "the father of all geometers."

We also find in this oration an early instance of the mistaken identification of Euclid the geometer with the philosophers Euclid of Megara mentioned in Plato's *Phaedo* (and by Diogenes Laërtius). This Renaissance mistake may go back to Theodoros Melichita in the early fourteenth century (Heath 1926, 3) (there is no reason to believe Theodoros to have been inspired by a similar but oblique reference in Valerius Maximus nor to assume that the fifteenth-century Italian writers took over from Valerius something that medieval authors, eager readers of his, had not thought of). But Latin humanist readers, finding the name Euclid in Plato, can also have reinvented the mistake independently.

Around the same time, two encyclopedic treatises coming from the Florentine abacus school environment testify of interest in the *Elements*. One is the anonymous Florence, Biblioteca Nazionale Centrale, Palat. 573; the other is Benedetto da Firenze's *Trattato di praticha d'arismetrica* from 1463, the autograph of which is Siena, L.IV.21. Both are described with copious extracts in Arrighi (2004). That they mention Euclid's name is not very informative – many abacus books excelled in dropping names in the way described by Boccaccio, often in fully misguided ways. But these two treatises are different: they refer repeatedly correctly to Euclidean books (II, V, VII, IX, X), at times with quotations pointing (as could be expected) to Campanus. The citations are so similar that the two writers must be assumed to have drawn on the same intermediate source – which shows that their branch of abacus culture (descending from Paolo dell'Abbaco and Antonio de' Mazzinghi) had a tradition for such interest in Euclid. How far this tradition goes back we cannot know, but it seems likely that it was a product of the fifteenth century. From the fifteenth century, there are also two manuscripts (Siena, L.IV.16 and 17) containing Italian translations of the Campanus version (Kristeller 1965, VI, 158; Folkerts 2006, XI 223), which might come from the same tradition.

Euclid in Print

The fifteenth century is also the time when some pure-bred humanists started collecting mathematical manuscripts for their libraries (Alberti, though a humanist, was more than that and so was Regiomontanus) – most famously of all probably Bessarion. However, Euclid was not their first choice. Accordingly, the first printed *Elements* (Campanus 1482) were made by Erhard Ratdolt in Venice in Campanus' version. Ratdolt was no humanist but an outstanding and innovative German printer. His dedicatory letter to Duke Mocenigo of Venice shows him to be more closely linked to the university tradition than to the humanist current.

This edition can be seen to have fulfilled a need. Ratdolt produced a reprint in the same year, another one was made in Ulm in 1486, and a third was in Basel in 1491 (Cantor 1892, 266*f*).

The first Euclidean texts produced by a humanist are found in an anthology collected by Giorgio Valla (1498, c vi^{r}–d v^{r}) – namely, *Elements* XIV and XV – and thus actually pseudo-Euclidean; Valla presents the former as Euclid's 14th book and the latter as Hypsicles's interpretation of the same book. Scattered properly Euclidean fragments taken from a Greek manuscript were to be found in his posthumous encyclopedia *De expetendis et fugiendis rebus opus* (Valla 1501). In this work he draws much on Proclus's commentary to *Elements* I, which gives him access to Eudemus's catalogue of geometers and to the dating of Euclid to the time of the first Ptolemy; nonetheless, the identification of the geometer and the philosopher from Megara survived not only Valla's insight but also the publication of Proclus's commentary in 1533 (*infra*).

The first full Renaissance text of the *Elements* translated from the Greek was produced by Bartolomeo Zamberti in 1505 (reprinted in 1510 and 1517 by the same Venetian printer), with copious attacks on Campanus. The (unidentified) manuscript he used was in the Theonine tradition, and Zamberti supposed that the demonstrations were due to Theon; Campanus instead had relied through Adelard on Arabic, pre-Theonine manuscripts, which is one of several reasons for the

divergences which provoked Zamberti's anger. The volume also contained Books XIV–XV, presented as "Hypsicles report of a volume supposed to be by Euclid," respectively, Book XIV of Euclid's *Elements* "in the report of Hypsicles" (Zamberti 1505, X iiii^{r}, vii^{v}); moreover, Euclid's *Phenomena*; the pseudo-Euclidean *Catoptrics*; Euclid's *Optics*; and his *Data*, with Marinus's introduction. A long dedication also serves as introduction, presenting the history of mathematics in its relation to philosophy from Homer to Plotinus and Proclus; where relevant, it draws on Proclus's commentary (and thus on Eudemus). Identifying Euclid with Euclid of Megara, Zamberti states him to have listened to Socrates and to be a contemporary of Plato and at the same time takes Proclus as his witness that Euclid lived at the time of the first Ptolemy. After the introduction comes Euclid's *vita*, consisting of excerpts from Suidas, Diogenes Laërtius, Plutarch, Aulus Gellius (all speaking of Euclid of Megara), Heron of Alexandria, Proclus, and Marinus (speaking of the mathematician). In the end, Zamberti attempts to find from available chronicles the epoch of Ptolemy I, coming to 291 BC; he does nothing to determine that of Socrates and appears not to be aware of any contradiction. In any case, this *vita* demonstrates to the full Zamberti's humanist credentials. Unfortunately, as Maurolico was to observe in a letter from 1556 (Napoli 1876, 27), while translating faithfully, Zamberti lacked the necessary mathematical insight and did not discover the mistakes of his manuscript.

Zamberti's attacks against Campanus were answered by Luca Pacioli, who produced a new edition of the Campanus *Elements* (Pacioli 1509), also in Venice. It promised to correct the errors that had crept in because of the negligence of copyists. Actually, Pacioli's text and his diagrams are very close to those of Ratdolt's edition, but he added a number of commentaries (Folkerts 2006, XI, 227*f*) – 138 in total, of which 42 over 10 lines long, according to Folkerts. Already a decade or so before publishing this edition, Pacioli had made a vernacular translation of Campanus, which however has been lost.

A phase followed where publishers might play safe and print the Campanus- and the Zamberti- texts together. The first edition of this kind was made by Jacques Lefèvre d'Étaples in 1516, containing only the *Elements* (15 books). Lefèvre d'Étaples supposes only the definitions, postulates, and common notions to be due to Euclid, while he ascribes the proofs to Campanus, respectively, Zamberti. Another combined edition was made by Herwagen in Basel in 1537 (Euclidis Megarensis 1537), with an introduction where Philip Melanchthon speaks at length about the utility and the moral implications of mathematics. It included the other Euclidean works translated by Zamberti (the *Data* still with Marinus's introduction) and the pseudo-Euclidean *De levi et ponderoso* (sic; *supra*), presented as a fragment and indeed lacking the fifth and final proposition of the medieval text (and formulated in very different words). New editions of this collection were published by Herwagen in 1546 and 1558 with only modest changes.

Already in 1533, Simon Grynaeus had brought out the *editio princeps*, also at the printing house of Herwagen – unfortunately from two low-quality manuscripts (Heath 1926, 101). It also contains Proclus's commentary to *Elements* I. Books XIV and XV are presented as "according to others, by Hypsicles." There is no trace of Euclid of Megara, not even a polemical refutation.

The number of editions and reprints may seem to suggest that there was strong interest at least in the *Elements*. A letter from Maurolico to Pietro Bembo (ed. Spezi 1862, 80) gives the opposite impression – "Galen flourishes everywhere, the academies resound with Justinian, the marble is shattered in dialectical disputes. Why is the excellent Euclid silent? Why are Archimedes and Theodosius silent? [. . .] Of Euclid, hardly six books are read." Compared to what we have encountered in fourteenth- and even fifteenth-century Humanism, however, "hardly six books" constitute a qualitative jump – and six books remained a standard school book for long.

The first (bilingual) six-book edition was published in the same year by Oronce Finé (Finé 1536). It presents itself as Finé's "demonstrations of the first six books of the *Elements* of Euclid of Megara, augmented and emended, together with the same Euclid's Greek text, and Zamberti's

Latin interpretation," all examined by Finé himself. What this means is that the part of the text which Zamberti had ascribed to Euclid – that is, definitions, postulates, common notions, and enunciations – are given as by Grynaeus and Zamberti. The demonstrations are rephrased pedagogically by Finé (and marked "Orontius").

A particular six-book version of "Euclides Megarensis" was published by Johann Scheubel (1550). It also borrows the formulation of the matters which were regarded as properly Euclidean from Grynaeus and Zamberti. The proofs, however, are not only Scheubel's own – the diagrams are not lettered; instead the lines often carry numbers corresponding to a supposed length; there are also regularly numerical computations – and the whole is preceded by a 76-page-long introduction to algebra (including the theory of irrationals). Rather than a humanist attempt to restore Greek mathematics, this edition is thus an early, still groping attempt at synthesis of Greek theory with the higher level of *Rechenmeister* mathematics.

A number of other editions based (sometimes faithfully, sometimes with innovations) on Grynaeus, Campanus, and Zamberti are listed in Heath (1926, 101*f*) and Murdoch (1971, 449). Taken together they confirm that there was much more interest in Euclid in the sixteenth than in previous centuries – Maurolico's complaints notwithstanding.

Further confirmation comes from the vernacular translations of the *Elements* that appeared; at the same time, these make clear the diversity of groups that partook in this interest. Earliest was (Tartaglia 1543), "according to the two translations" (Campanus and Zamberti) but presenting a single text. Different from Finé and the combined editions of Lefèvre d'Étaples and Herwagen, Tartaglia presents enunciations and demonstrations as belonging on an equal footing to the *Elements* and adds his own discussions under the heading "The translator."

A French translation followed (Forcadel 1564) (six books only), in which the demonstrations are (justly) ascribed to Forcadel. Scheubel made a German translation of Books VII–IX in 1558; Xylander a full German translation adapted to the needs of artisans (that is, largely without proofs) in 1562 (Heath 1926, 106–108; Murdoch 1971, 449); and in 1570, Henry Billingsley made an impressive English translation, prefaced by John Dee, and including "Scholies, Annotations, and Inventions, of the Best Mathematiciens, both in time past, and in this our age" (Heath 1926, 109).

Euclid for the Future: Commandino and Clavius

All of these editions and translations, from Zamberti onward, can be said to define and even constitute the sixteenth-century Euclid. In the early 1570s, however, two very different translations appeared that were to define the Euclid of the next two centuries. One was Commandino (1572). Federico Commandino based this Latin translation on Grynaeus as well as another, better Greek manuscript (still in the Theonine tradition), and also included many previously unknown Greek scholia; moreover, being an outstanding mathematician, he understood the text much better than any sixteenth-century predecessor. There were thus very good reasons that Commandino's translation became the direct or indirect basis for many new translations and editions (Murdoch 1971, 44) – also editions combining with Grynaeus Greek text. It was superseded only when François Peyrard discovered and published an apparently pre-Theonine text (Peyrard 1804).

Commandino was also a better historian than his predecessors. Whereas Valla and Grynaeus appear not to have believed in the identification with Euclid of Megara but did not discuss why, and while Zamberti did not notice that the evidence he draws on excludes it, Commandino wants (fourth page of the unfoliated preface) to "free those many from their error who firmly believe our Euclid to be the same as both the philosopher from Megara and the geometer" and sets out briefly the reasons – both chronological and from what Diogenes Laërtios tells about the work of Euclid of Megara.

The other was Clavius (1574). In an initial address to the reader, Christophorus Clavius explains why all preceding editions are deficient –

"excepting that of Federico Commandino, a highly skillful geometer, by whose work and diligence Euclid has been rendered in Latin in its pristine splendor." However, Clavius's aim is (as he explains) to produce a book that can serve those who progress in the sweet study of mathematics; therefore he does not propose Euclid's naked words, which are often more concise than illuminating. He provides them with extra explanations, sometimes his own, sometimes borrowed from Proclus (Clavius uses Francesco Barozzi's translation from 1560), Campanus, or others. Rommevaux (2005, 31–58) analyzes the additions and occasional changes of the Euclidean text.

Clavius published new editions of these *Elements* in 1589, 1591, 1603, 1607, and 1612. They were used for the teaching of Jesuit recruits and hence became the book behind the teaching of much of the social elite of Catholic Europe for long. They were also the Euclid which Matteo Ricci and Xu Guangqi translated into Chinese in 1607 (the usual six books), in time for Clavius to see the Chinese printed book.

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